GLM with two fixed explanatory variables 3 examples Factor * Factor
Factor * Covariate
Covariate * Covariate

Format for writing a model with two explanatory variables

$Response = \beta_o + \beta_{V1}V1 + \beta_{V2}V2 + \beta_{V1\times V2}\, V1\times V2 + \varepsilon_{Normal}$

The interactive term is written as the product of two component variables $\beta_{V1\times V2}\, V1\times V2$

Verbal statement: The effect of V1 on the response variable depends on V2

Write the Fixed factor × Fixed factor GLM, calculate df, fill out the Source df table

df total = *ntot*-1 df $V1\times V2$ = df($V1$) × df($V2$)

4. Does oxygen consumption VO_2 depend on salinity (100% 75% and 50% seawater) in two species of limpet (*Acmea digitalis* and *A. scabra*)? Eight measurements at 3 different salinities in each of two species *ntot* = 48. Data from Sokal and Rohlf (1995).

Response variable with symbol ____________________

Explanatory variable	Symbol	Categorical or Ratio scale
____________	____	________
____________	____	________

Model ______________________________

df ______________________________

Source	df
	42

Interpret the interactive effect (state this in words)

http://www.mun.ca/biology/schneider/b4605/LNotes/Pt4/Ch13_1.pdf

http://www.mun.ca/biology/schneider/b4605/GLMMworkshop/Data/Limpets.csv

GLM with two fixed explanatory variables 3 examples Factor * Factor
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$2(8\times3) = 48$

Response variable with symbol Oxygen Consumption $\dot{V}O_2$

Explanatory variable	Symbol	Categorical or Ratio scale
Salinity	Sal	3 categories
Species	Sp	2 categories

Model $H_{zyg} = \beta_{Sal} Sal + \beta_{Sp} Sp + \beta_{Sal\cdot Sp} Sal\cdot Sp + \varepsilon$

df $47 = 2 + 1 + 2\times1 + 42$

Source	df
Sal	2
Sp	1
Sal × Sp	2
Error	42
total	47

Interpret the interactive effect (state this in words)

http://www.mun.ca/biology/schneider/b4605/LNotes/Pt4/Ch13_1.pdf

http://www.mun.ca/biology/schneider/b4605/GLMMworkshop/Data/Limpets.csv

GLM with two fixed explanatory variables

Factor * Factor
2nd example -> Factor * Covariate (aka ANCOVA)
Covariate * Covariate

5. Does inversion heterozygosity (*Hzyg*) change with elevation above sea level (*Hsl*), in 2 species of *Drosophila* (SP = *D. persimilis* or *D. pseudoobscura*). Data are from Dobzhansky (1948) as reported in Brussard (1984). One measurement in each species at 7 different elevations.

Source	df
	10

Model ______________________________

df ______________________________

Complete the Source df table.
Interpret the interactive effect (state it in words)

http://www.mun.ca/biology/schneider/b4605/LNotes/Pt4/Ch14_1.pdf

http://www.mun.ca/biology/schneider/b4605/GLMMworkshop/Data/Brussard.csv

GLM with two fixed explanatory variables

Factor * Factor
2nd example -> Factor * Covariate (aka ANCOVA)
Covariate * Covariate

5. Does inversion heterozygosity (*Hzyg*) change with elevation above sea level (*Hsl*), in 2 species of *Drosophila* (SP = *D. persimilis* or *D. pseudoobscura*). Data are from Dobzhansky (1948) as reported in Brussard (1984). One measurement in each species at 7 different elevations.

Source	df
	10

Model ______________________________

df ______________________________

Complete the Source df table.
Interpret the interactive effect (state it in words)

http://www.mun.ca/biology/schneider/b4605/LNotes/Pt4/Ch14_1.pdf

http://www.mun.ca/biology/schneider/b4605/GLMMworkshop/Data/Brussard.csv

GLM with two fixed explanatory variables

Factor * Factor
Factor * Covariate
3rd example -> Covariate * Covariate (aka multiple regression)

6. Data from Snedecor and Cochrane 1980 Table 17.2.1
Does plant available phosphorus content of corn (ppm) from 17 Iowa soils at 20 deg C depend on inorganic and organic phosphorus in the soil?

Source	df
	13

Model ________________________________

df ________________________________

Complete the Source df table.
Interpret the interactive effect (state it in words)

http://www.mun.ca/biology/schneider/b4605/LNotes/Pt4/Ch12_1.pdf

http://www.mun.ca/biology/schneider/b4605/GLMMworkshop/Data/PAvailable.csv

GLM with two fixed explanatory variables Factor * Factor
Factor * Covariate
3rd example -> Covariate * Covariate (aka multiple regression)

6. Data from Snedecor and Cochrane 1980 Table 17.2.1
Does plant available phosphorus content of corn (ppm) from 17 Iowa soils at 20 deg C depend on inorganic and organic phosphorus in the soil?

.

Source	df
	13

Model ____________________________________

df ____________________________________

Complete the Source df table.
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http://www.mun.ca/biology/schneider/b4605/LNotes/Pt4/Ch12_1.pdf

http://www.mun.ca/biology/schneider/b4605/GLMMworkshop/Data/PAvailable.csv

GLM - Random Effects. The first solution to heterogeneous errors.

The GLM assumes a normal error with fixed (constant) variance $= \varepsilon_{Normal}$
Grouped data usually violate this assumption.-- > heterogeneous residuals
Examples: Paired data, clustered data, blocked data
Examples: Repeated measures (*e.g.* 3 samples at one time), longitudinal data (3 samples in sequence)
To capture this heterogeneity, we introduce a random effect variable Z with random coefficients τ (tau).

$$Y = \mu_o + \tau_Z Z + \varepsilon_{Normal}$$

μ_o = random intercept
$\tau_Z Z$ = random effect

GLM Single Random Factor

10 The first published ANOVA table was Example 38 in Fisher (1925) *Statistical Methods for Research Workers.* "In an experiment on the accuracy of counting soil bacteria, a soil sample was divided into four parallel samples and from each of these after dilution seven plates were inoculated. The number of colonies on each plate is shown below in example 12 (Table 41). Do the results from the four samples agree within the limits of random sampling? In other words, is the whole set of 28 values homogeneous, or is there any perceptible intraclass correlation?"

Table 42	Degrees of Freedom	Sum of Squares	Mean Square	F-ratio	R^2	Likelihood Ratio
Between Classes (Soil sample)	3	1446				
Within Classes (Error)	24	94.96				

Assign a symbol to the response variable _____ and explanatory variable _________

Using the notation shown above, write the model (use μ and τ) ________________________________

Compute both mean squares (= SS/df) and place them in the ANOVA table
Compute the ratio of the two means squares (the F-ratio) and place it in the table
Compute the explained variance R^2 = Between class SS/SS_{total} = ________________
Do the 4 samples deviate from random sampling? To find out we calculate the likelihood ratio.

$LR = (1\text{-}R^2)^{-n/2}$ = ________________

Likelihood Ratio test: Compare the F-ratio to the 5% p-value of the F-distribution
The 5% probability for the F-distribution (excel code) is: FINV(0.05,3,24) = 3.009
Do the results from the four samples agree within the limits of random sampling? _______
http://www.mun.ca/biology/schneider/b4605/GLMMworkshop/Data/FisherEx38.csv

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GLM with two random factors 2 examples

Nested - Random within Random
Crossed - Random × Random

$$Y = \mu_o + \Sigma\tau_Z Z + \varepsilon_{Normal}$$
$$\Sigma\tau_Z Z = \text{sum of random effects of random variable } Z$$

11. Winglength of 12 mosquitos (3 cages, 4 flies per cage). The left wing of each fly was measured twice.

Source	df	SS	MS	F	---->	p
Cage	2	665.68	332.84	1.74		0.23
Fly⊂Cage	9	1720.68	191.19	147.07		<0.0001
Error	12	15.62	1.3017			
Total	23	2401.97				

ANOVA table
Table 10.1 in Sokal and Rohlf (1995).

Write the model from the Source and df columns in the ANOVA table

____

Show how each df was calculated: 2 = ________ 9 = __________

23 = __________ 12 = __________

Note that the Cage F-ratio was not calculated with respect to the MS error.
The Cage F-ratio was calculated from a random factor, Fly(Cage). Why ? Stay tuned.

http://www.mun.ca/biology/schneider/b4605/LNotes/Pt4/Ch13_6.pdf

http://www.mun.ca/biology/schneider/b4605/GLMMworkshop/Data/FisherEx38.csv

GLM with two random factors 2 examples

Nested - Random within Random
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Error	12	15.62	1.3017			
Total	23	2401.97				

ANOVA table
Table 10.1 in Sokal and Rohlf (1995).

Write the model from the Source and df columns in the ANOVA table

$WL = \beta_0 + \tau_C C + \tau_F Fly(Cage) + error$

Show how each df was calculated: 2 = 3-1 9 = 3(4-1)

23 = 24-1 12 = 23-2-9

Note that the Cage F-ratio was not calculated with respect to the MS error.
The Cage F-ratio was calculated from a random factor, Fly(Cage). Why ? Stay tuned.

http://www.mun.ca/biology/schneider/b4605/LNotes/Pt4/Ch13_6.pdf

http://www.mun.ca/biology/schneider/b4605/GLMMworkshop/Data/FisherEx38.csv

GLM with two random factors

2nd example - - >

Nested - Random within Random
Crossed - Random × Random

12. Fisher's Table 42 (Example 38) shows a nested design.

Plate		Sample		
	I	II	III	IV
1	72	74	78	69
2	69	72	74	67
3	63	70	70	66
4	59	69	58	64
5	59	66	58	62
6	53	58	56	58
7	51	52	56	54
Total	426	461	450	440
Mean	60.86	65.86	64.29	62.86

It ignores the fact that each plate was inoculated with subsamples from each of the four initial samples (Classes). Consequently, we can treat class (*i.e.* sample) as a random factor with 4 levels and cross it with another random factor, plate.
Assign symbols to both explanatory variables and write a two way random effects GLM with an interaction term.

Symbols ____

Model ____

Complete the Source and df columns of the ANOVA table for this model.
The correct model is a saturated model, the error term will have zero degrees of freedom.
We'll use this in the next session.

GLM with two random factors

2nd example - - >

Nested - Random within Random
Crossed - Random × Random

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Model ____

Complete the Source and df columns of the ANOVA table for this model.
The correct model is a saturated model, the error term will have zero degrees of freedom.
We'll use this in the next session.

Random or Fixed? The definition of fixed versus random differs among text books.

Definition from Quinn and Keough (2002)

There are two types of categorical predictor variables in linear models. The most common type is a fixed factor, where all the levels of the factor (*i.e.* all the groups or treatments) that are of interest are included in the analysis. We cannot extrapolate our statistical conclusions beyond these specific levels to other groups or treatments not in the study. If we repeated the study, we would usually use the same levels of the fixed factor again. Linear models based on fixed categorical predictor variables (fixed factors) are termed fixed effects models (or Model 1 ANOVAs). Fixed effect models are analogous to linear regression models where X is assumed to be fixed.
The other type of factor is a random factor, where we are only using a random selection of all the possible levels (or groups) of the factor and we usually wish to make inferences about all the possible groups from our sample of groups. If we repeated the study, we would usually take another sample of groups from the population of possible groups.

Drawing a branching tree diagram is not a reliable way to distinguish crossed from nested designs.
Why? Because a crossed design can be drawn as a branching tree.
The reliable way to distinguish crossed and nested designs is to write all of the two way tables and fill in the sample size in each cell of each table. If all (or most) of the cells have at least one sample then the two variables are crossed. If not the two factors are nested. For three factors there are three pairs and so three two-way tables.

GLMM with two explanatory variables 2 examples Fixed + Random
Fixed × Random

The GLM assumes a normal error with fixed (constant) variance $= \varepsilon_{Normal}$
Grouped data often violate this assumption.-- > heterogeneous residuals

Paired data, clustered data, blocked data

Repeated measures (e.g. 3 samples at once), longitudinal data (3 sequential samples)

To capture this heterogeneity, we write a General Linear Mixed Model, which has both fixed and random effects.

$$Y = \beta_o + \Sigma\beta_X X + \Sigma\tau_Z Z + \varepsilon_{Normal}$$

$$\Sigma\beta_X X = \text{sum of fixed effects}$$

$$\Sigma\tau_Z Z = \text{sum of heterogeneous random effects}$$

$$\varepsilon_{Normal} = \text{homogeneous normal errors}$$

GLMM with two explanatory variables First example Random(Fixed) Wheat Yields

13. Wheat Yields from Cornell (1971)

Three pots were assigned to each treatment.

The two-way (Pot × Treatment) table now has 12 cells.

There is 1 sample in each cell.

When we do the cross test the design appears to be crossed.

However, there were 12 pots in the experiment, not 3.

Treatment	Pot Number	Plant number 1	2	3
None	1	20.6	22.3	19.8
None	2	23.4	21.9	22.8
None	3	21.8	20.6	21.3
Straw	1	13.6	13.9	14.2
Straw	2	13.7	14.5	13.8
Straw	3	12.9	13.1	13.4
Straw + PO4	1	14.8	14.6	14.9
Straw + PO4	2	14.3	13.9	13.5
Straw + PO4	3	14.4	13.8	14.1
Straw+PO4+lime	1	14.1	13.8	14.3
Straw+PO4+lime	2	14.0	13.9	14.2
Straw+PO4+lime	3	14.4	14.1	13.6

http://www.mun.ca/biology/schneider/b4605/GLMMworkshop/Data/WheatYield.csv

Recode the Pot variable to show that there are 12 pots.

The two-way (Pot × Treatment) table now has 36 cells.

Most of the cells are empty.

We cannot estimate Pot × Treatment.

Pot is nested within treatment Pot(Treatment)

Carry out the cross test for Pot × Plant and Trt × Plant.

Now many cells? ________________

How many empty cells? ______________

Can Pot × Plant be estimated ? Y/N ____________

Can Trt × Plant be estimated ? Y/N ____________

Treatment	Pot Number	Plant number 1	2	3
None	1	20.6	22.3	19.8
None	2	23.4	21.9	22.8
None	3	21.8	20.6	21.3
Straw	4	13.6	13.9	14.2
Straw	5	13.7	14.5	13.8
Straw	6	12.9	13.1	13.4
Straw + PO4	7	14.8	14.6	14.9
Straw + PO4	8	14.3	13.9	13.5
Straw + PO4	9	14.4	13.8	14.1
Straw+PO4+lime	10	14.1	13.8	14.3
Straw+PO4+lime	11	14.0	13.9	14.2
Straw+PO4+lime	12	14.4	14.1	13.6

GLMM with two explanatory variables First example Random(Fixed) Wheat Yields

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Straw	2	13.7	14.5	13.8
Straw	3	12.9	13.1	13.4
Straw + PO4	1	14.8	14.6	14.9
Straw + PO4	2	14.3	13.9	13.5
Straw + PO4	3	14.4	13.8	14.1
Straw+PO4+lime	1	14.1	13.8	14.3
Straw+PO4+lime	2	14.0	13.9	14.2
Straw+PO4+lime	3	14.4	14.1	13.6

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Can Trt × Plant be estimated ? Y/N ______________

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None	3	21.8	20.6	21.3
Straw	4	13.6	13.9	14.2
Straw	5	13.7	14.5	13.8
Straw	6	12.9	13.1	13.4
Straw + PO4	7	14.8	14.6	14.9
Straw + PO4	8	14.3	13.9	13.5
Straw + PO4	9	14.4	13.8	14.1
Straw+PO4+lime	10	14.1	13.8	14.3
Straw+PO4+lime	11	14.0	13.9	14.2
Straw+PO4+lime	12	14.4	14.1	13.6

GLMM with two explanatory variables 2nd example Fixed × Random

Subject	Drug A	Drug B
1	0.7	1.9
2	-1.6	0.8
3	-0.2	1.1
4	-1.2	0.1
5	-0.1	-0.1
6	3.4	4.4
7	3.7	5.5
8	0.8	1.6
9	0.0	4.6
10	2.0	3.4

14. Sleep data (Cushny and Peebles), used by Student (W. Gossett) to introduce the *t*-test. Data are: hours of extra sleep with two drugs Hyoscyamine (Drug A) and L Hyoscine (Drug B), each administered to 10 subjects. Values reported are averages. The pairing across subject allows us to remove the effects of individual variation.

Assign a symbol to the response variable ______________

For each explanatory variable assign a symbol and state reason for assigning it as Fixed or Random

________ ______________________________________

________ ______________________________________

http://www.mun.ca/biology/schneider/b4605/LNotes/Pt4/Ch13_3.pdf

http://www.mun.ca/biology/schneider/b4605/GLMMworkshop/Data/ExtraSleep.csv

Crossed or Nested?

There are only two variables, hence only one interaction term.
We can see right away that this is a crossed design.

GLMM with two explanatory variables 2nd example Fixed × Random

Subject	Drug A	Drug B
1	0.7	1.9
2	-1.6	0.8
3	-0.2	1.1
4	-1.2	0.1
5	-0.1	-0.1
6	3.4	4.4
7	3.7	5.5
8	0.8	1.6
9	0.0	4.6
10	2.0	3.4

14. Sleep data (Cushny and Peebles), used by Student (W. Gossett) to introduce the *t*-test. Data are: hours of extra sleep with two drugs Hyoscyamine (Drug A) and L Hyoscine (Drug B), each administered to 10 subjects. Values reported are averages. The pairing across subject allows us to remove the effects of individual variation.

Assign a symbol to the response variable Hr

For each explanatory variable assign a symbol and state reason for assigning it as Fixed or Random

Drug Fixed : Infer only to Drug A + B

Subj Random : Infer to other subjects

http://www.mun.ca/biology/schneider/b4605/LNotes/Pt4/Ch13_3.pdf

http://www.mun.ca/biology/schneider/b4605/GLMMworkshop/Data/ExtraSleep.csv

Crossed or Nested?

There are only two variables, hence only one interaction term.
We can see right away that this is a crossed design. ✓